دانشگاه سیستان و بلوچستان

Signals and Systems
FFT Algorithm and Its Applications

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Discrete Fourier Transform Computational Complexity

$$\begin{cases} F[n] = \sum_{k=<N>} f[k]\, e^{-j\frac{2\pi}{N}nk}, (n = 0 :\ N-1) \\ W = e^{-j\frac{2\pi}{N}} \end{cases} \rightarrow F[n] = \sum_{n=<N>} f[k]\, W^{nk}$$

$$\begin{bmatrix} f[0] \\ f[1] \\ f[2] \\ f[3] \\ \vdots \\ f[N-1] \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & W & W^2 & W^3 & \cdots & W^{N-1} \\ 1 & W^2 & W^4 & W^6 & \cdots & W^{N-2} \\ 1 & W^3 & W^6 & W^9 & \cdots & W^{N-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W^{N-1} & W^{N-2} & W^{N-3} & \cdots & W \end{bmatrix} = \begin{bmatrix} F[0] \\ F[1] \\ F[2] \\ F[3] \\ \vdots \\ F[N-1] \end{bmatrix}$$

$O(n^2)$

Fast Fourier Transform (FFT) Algorithm

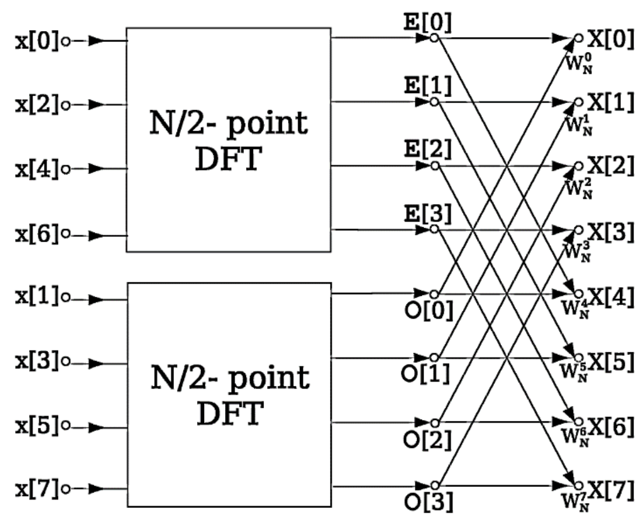
$$F[n] = \sum_{k=<N>} f[k]\, W_N^{nk}$$

$$= \sum_{m=0}^{\frac{N}{2}-1} f[2m]\, W_N^{2mn} + \sum_{m=0}^{\frac{N}{2}-1} f[2m+1]\, W_N^{(2m+1)n}$$

$$= \sum_{m=0}^{\frac{N}{2}-1} f[2m]\, W_{\frac{N}{2}}^{mn} + W_N^{m} \sum_{m=0}^{\frac{N}{2}-1} f[2m+1] W_{\frac{N}{2}}^{mn}$$

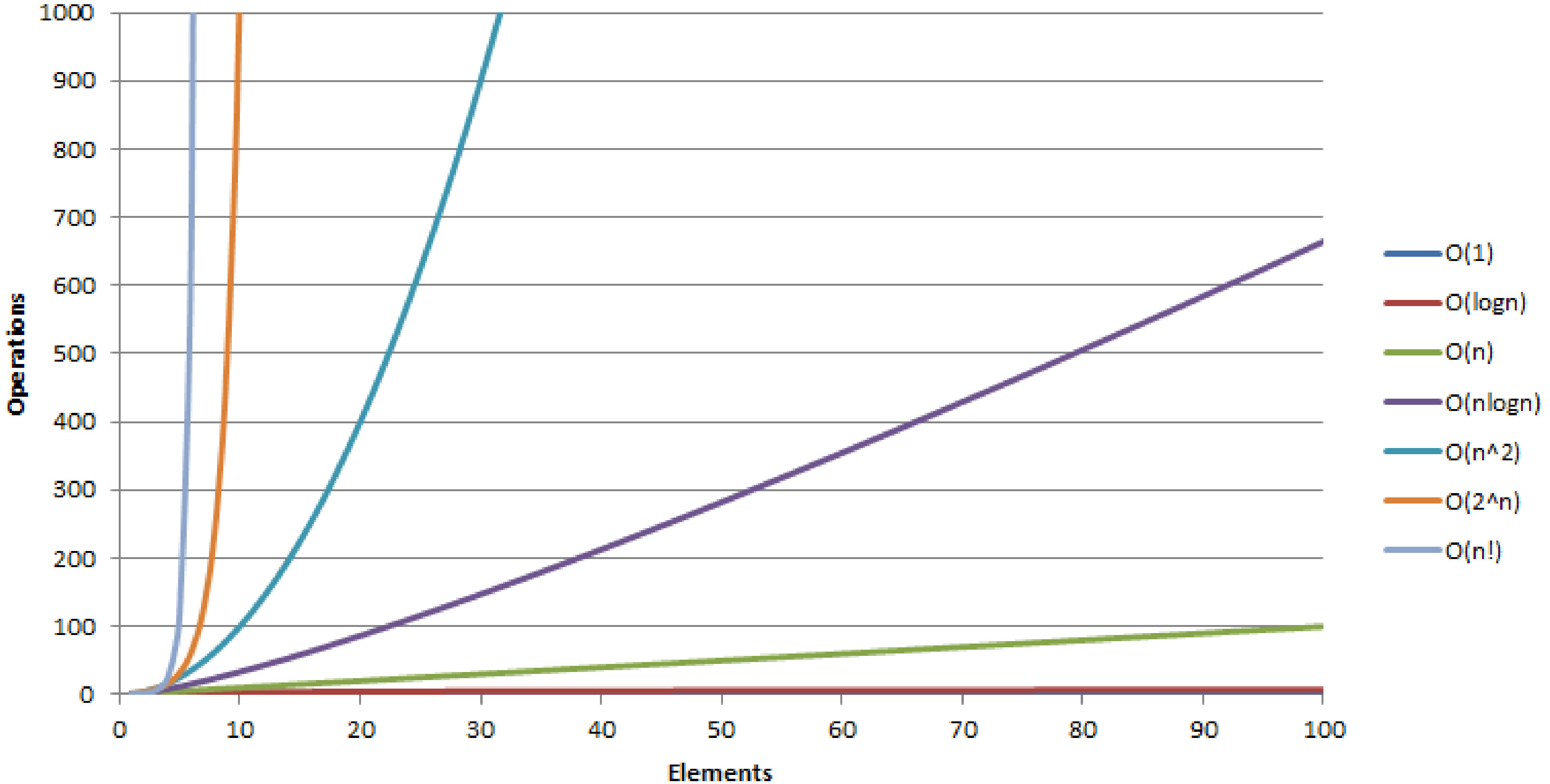
$$= \mathrm{G[n]} + W_N^{m} H[n]$$

$$\rightarrow Sum\ of\ 2\ \frac{N}{2}\ point\ DFTs\ (divide\ and\ conquer)$$



$$O(\mathrm{nlog}_2\, n)$$

Time Complexity Graph

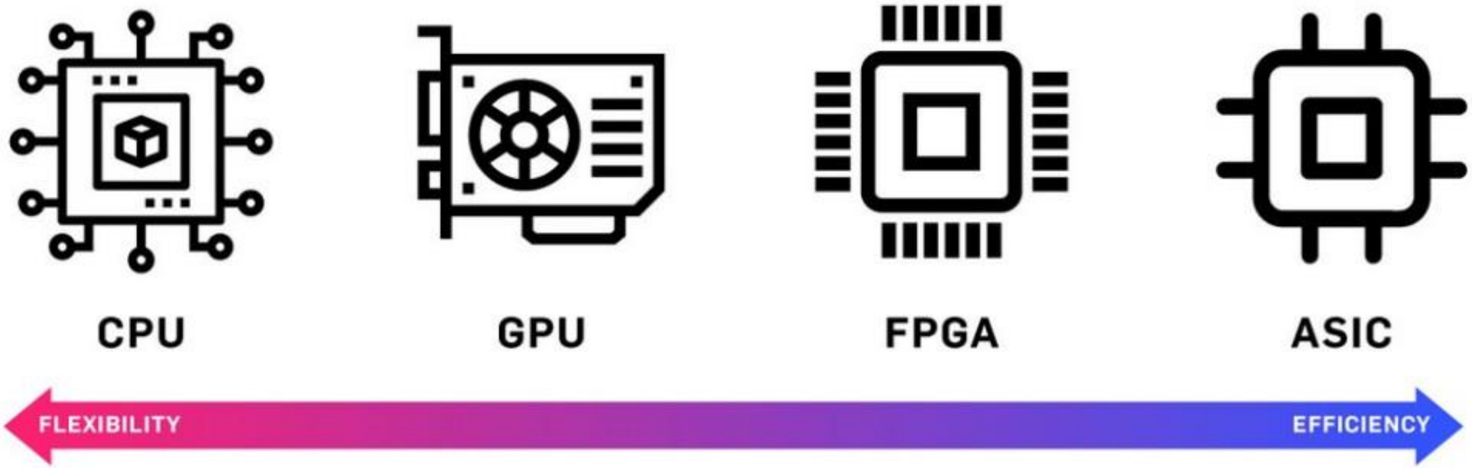


For 10 seconds of an audio at 44 kHz sampling rate

$O(n^2) \rightarrow 10^{11}$ *multiplications*

$O(n \log_2 n) \rightarrow 10^6$ *multiplications*

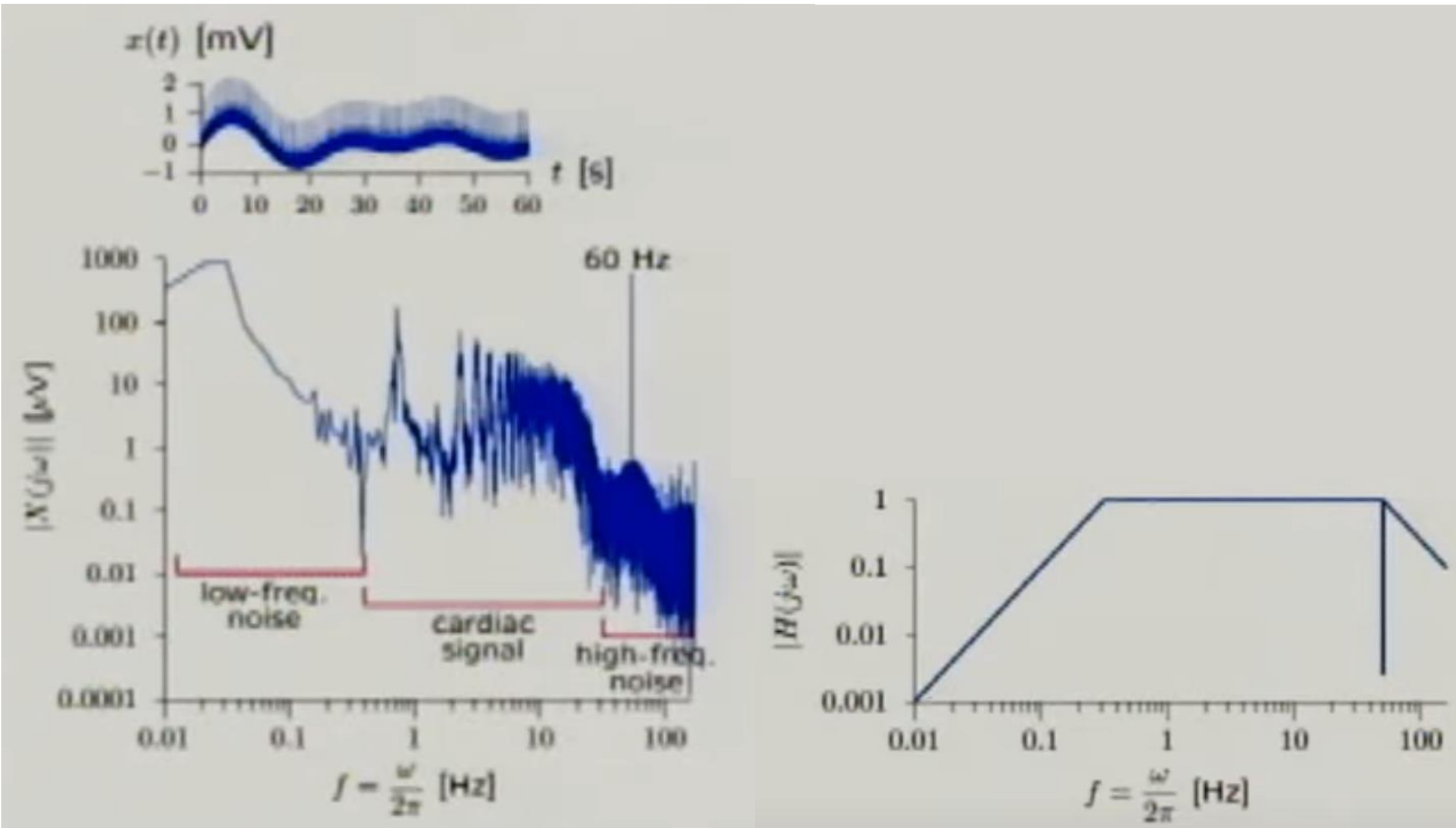
FPGA and ASICs



FFT Applications

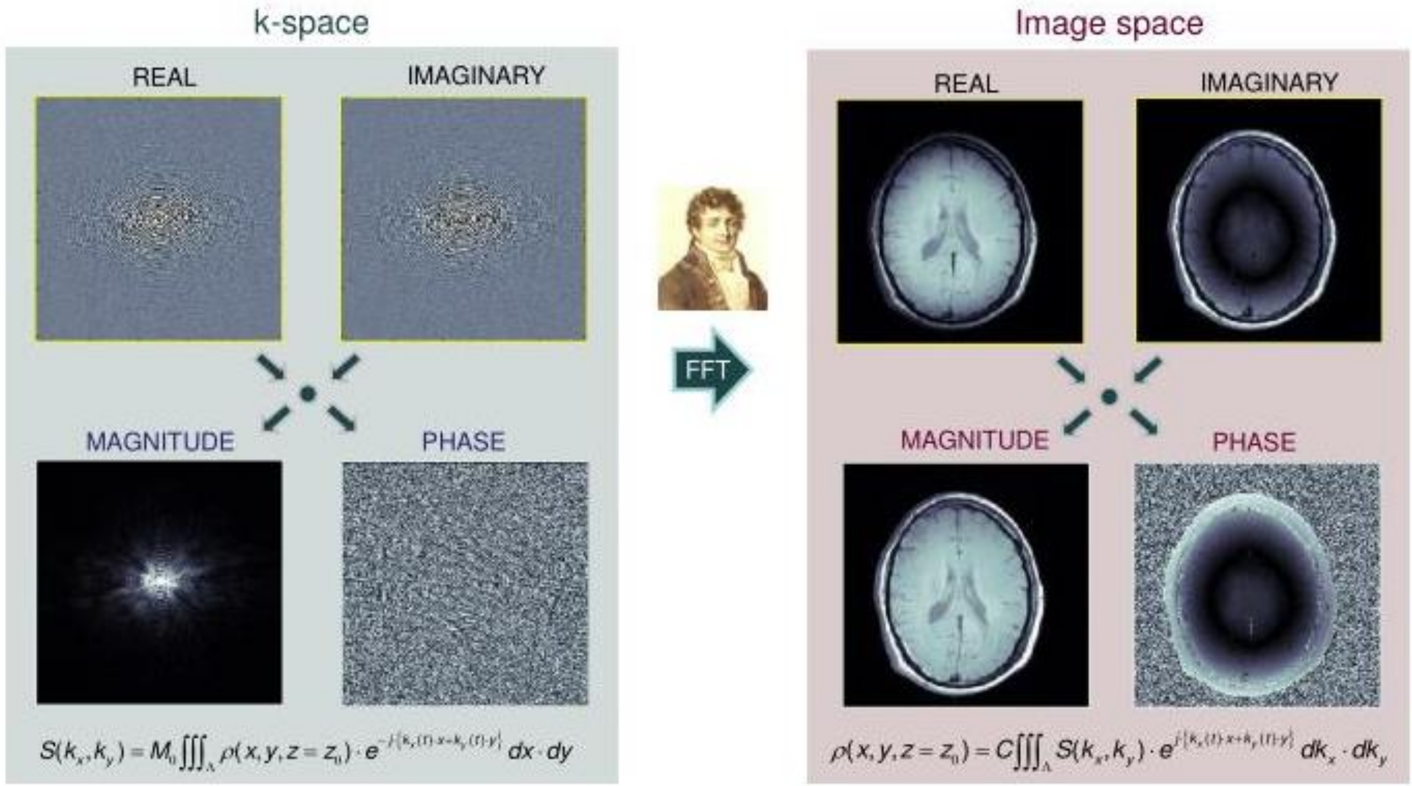
1. Electrocardiography (ECG)
2. Magnetic Resonance Imaging (MRI)
3. Edge detection in image processing
4. Orthogonal Frequency Division Multiplexing (OFDM)
5. JPEG and MPEG/MP3 encoding and decoding

FFT and ECG



Picture Source: Freeman, D. (2011). Filtering Example: Electrocardiogram. MIT 6.003 Signals and Systems.

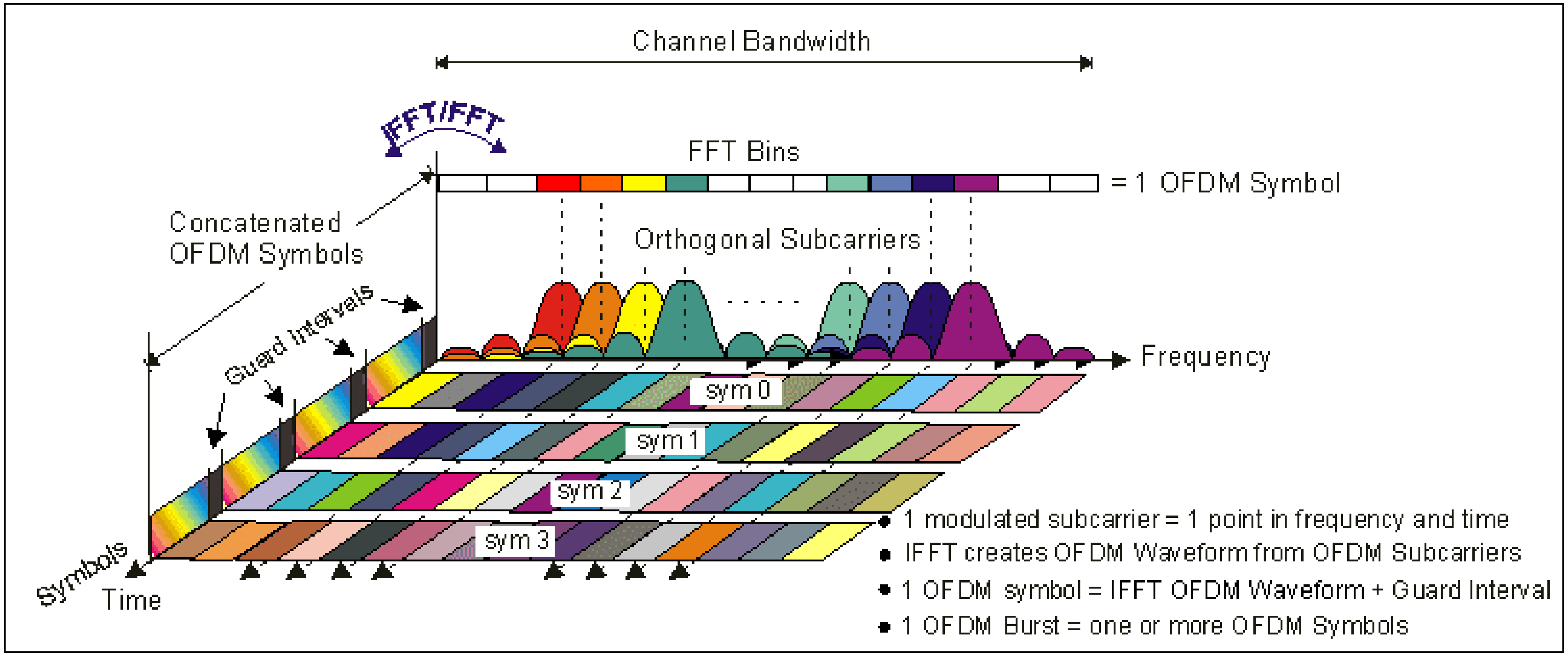
FFT and MRI



Picture Source: Deene, Y. D. Basic MRI principles. Research Group Quantitative MRI In Medicine and Biology. Universiteit Gent

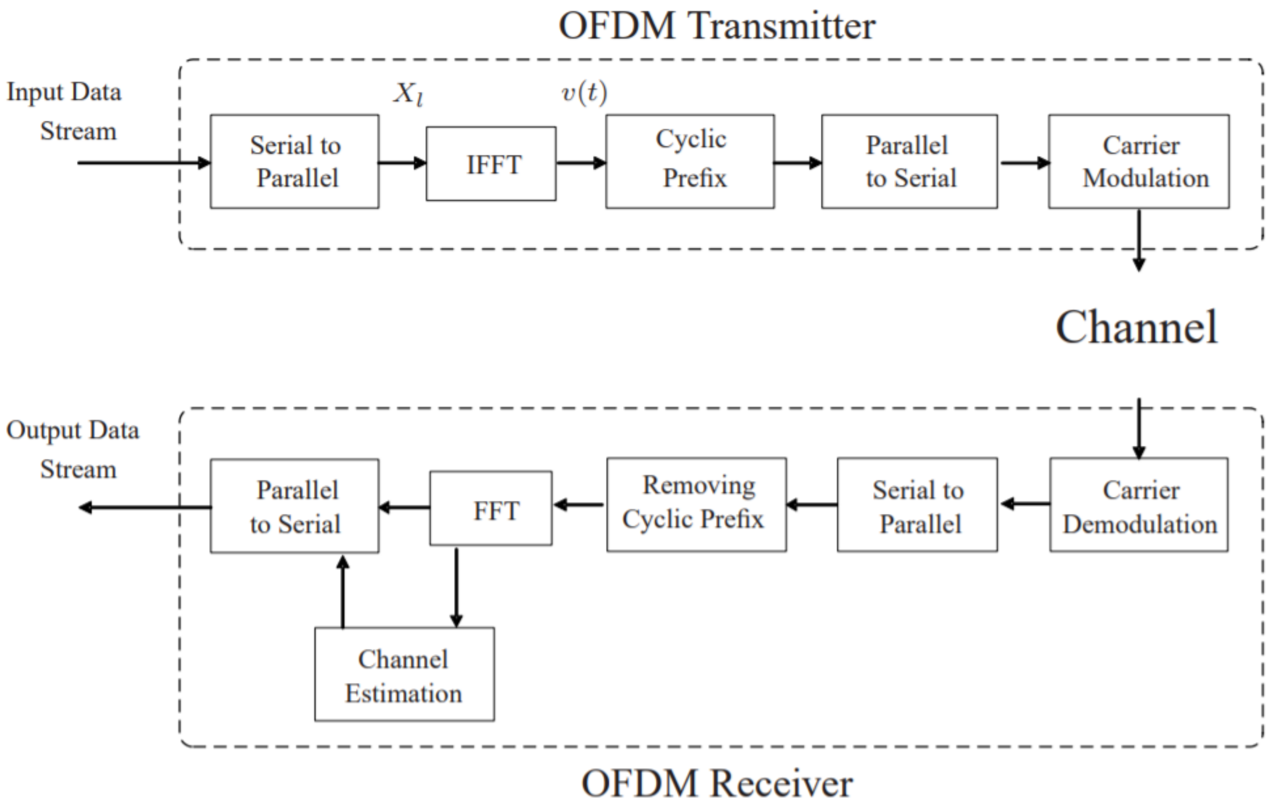
FFT and Edge Detection

FFT and OFDM



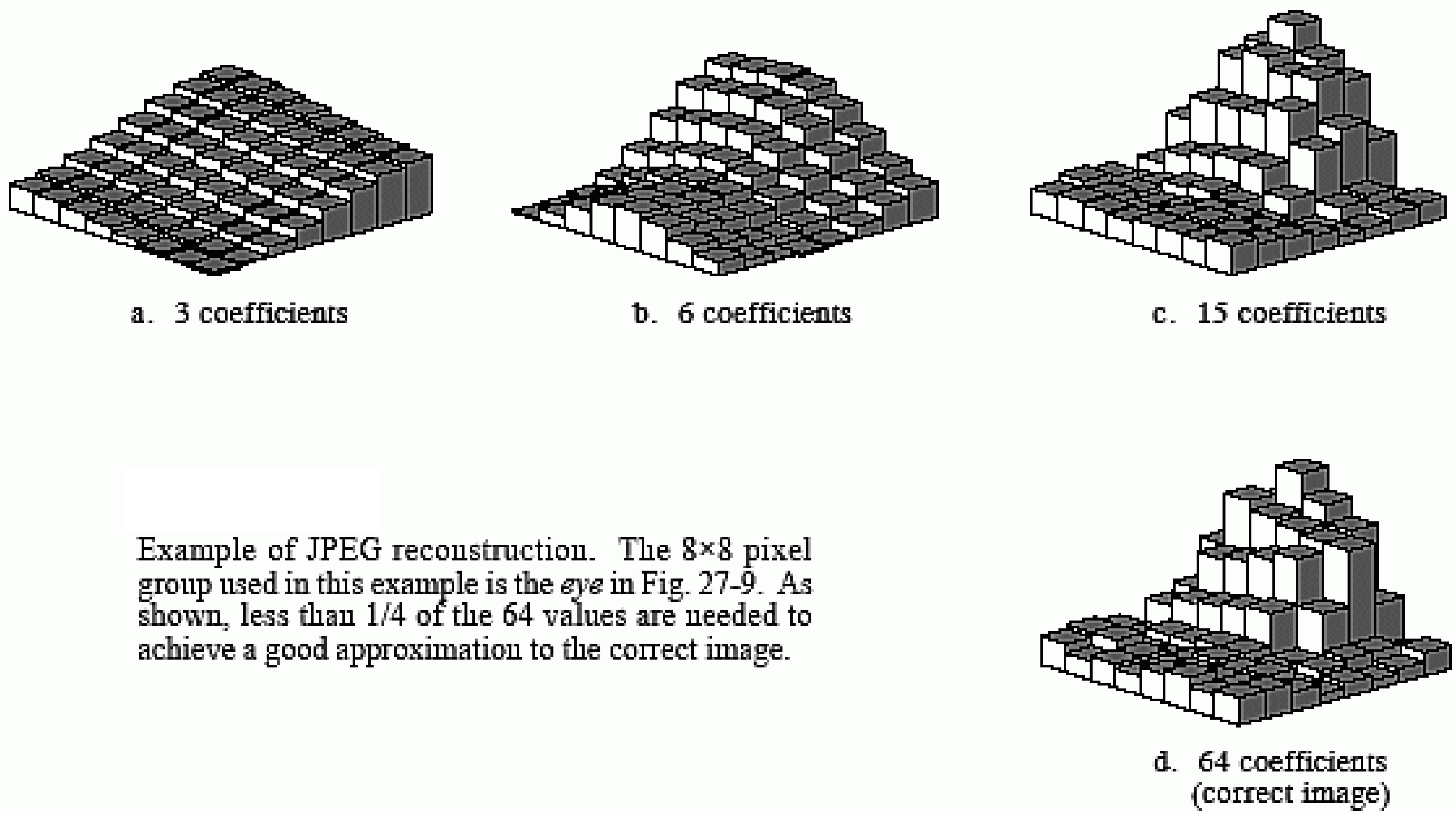
Frequency-Time Representative of an OFDM signal

FFT and OFDM (cont.)



Picture Source: Mehri Mehrjoo (2008). Resource Allocation in OFDMA Wireless Networks. UWSpace. 🙂

FFT and JPEG



Example of JPEG reconstruction. The 8×8 pixel group used in this example is the *eye* in Fig. 27-9. As shown, less than 1/4 of the 64 values are needed to achieve a good approximation to the correct image.

The END!