دانشگاه سیستان و بلوچستان

Signals and Systems
Laplace Transform and Its Applications

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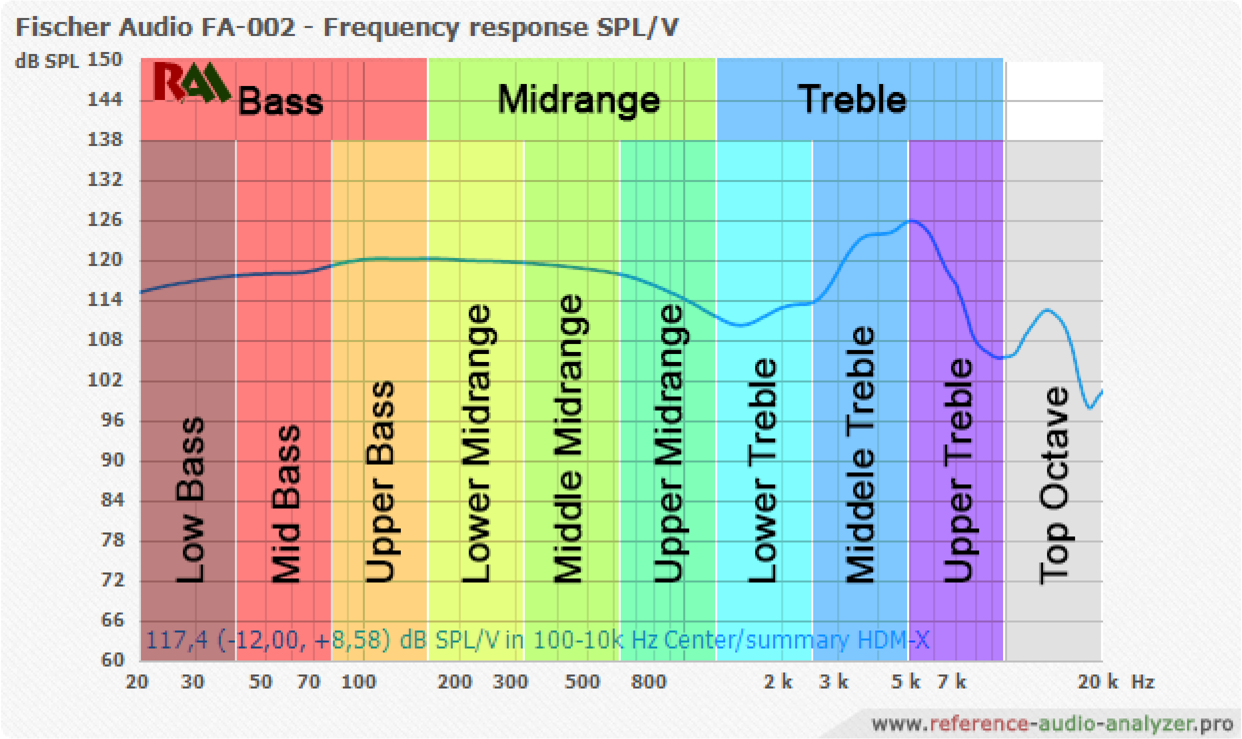
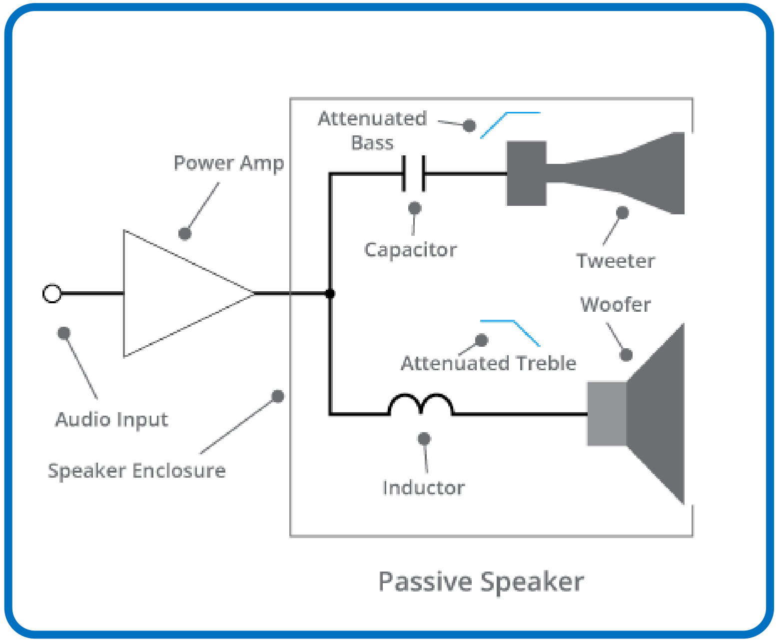
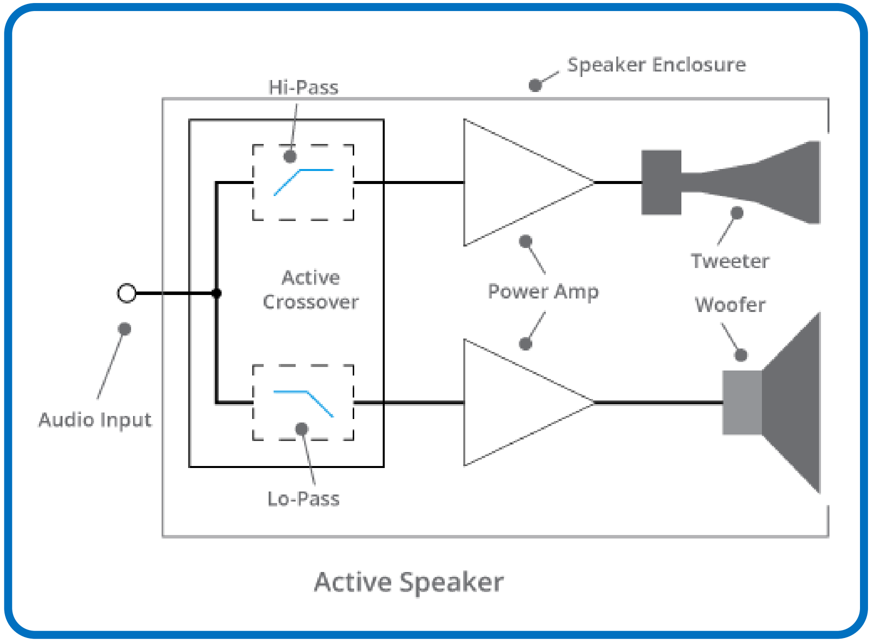
Frequency Range



Note: This slide has been used in assignment 9 to present applications of Fourier analysis.

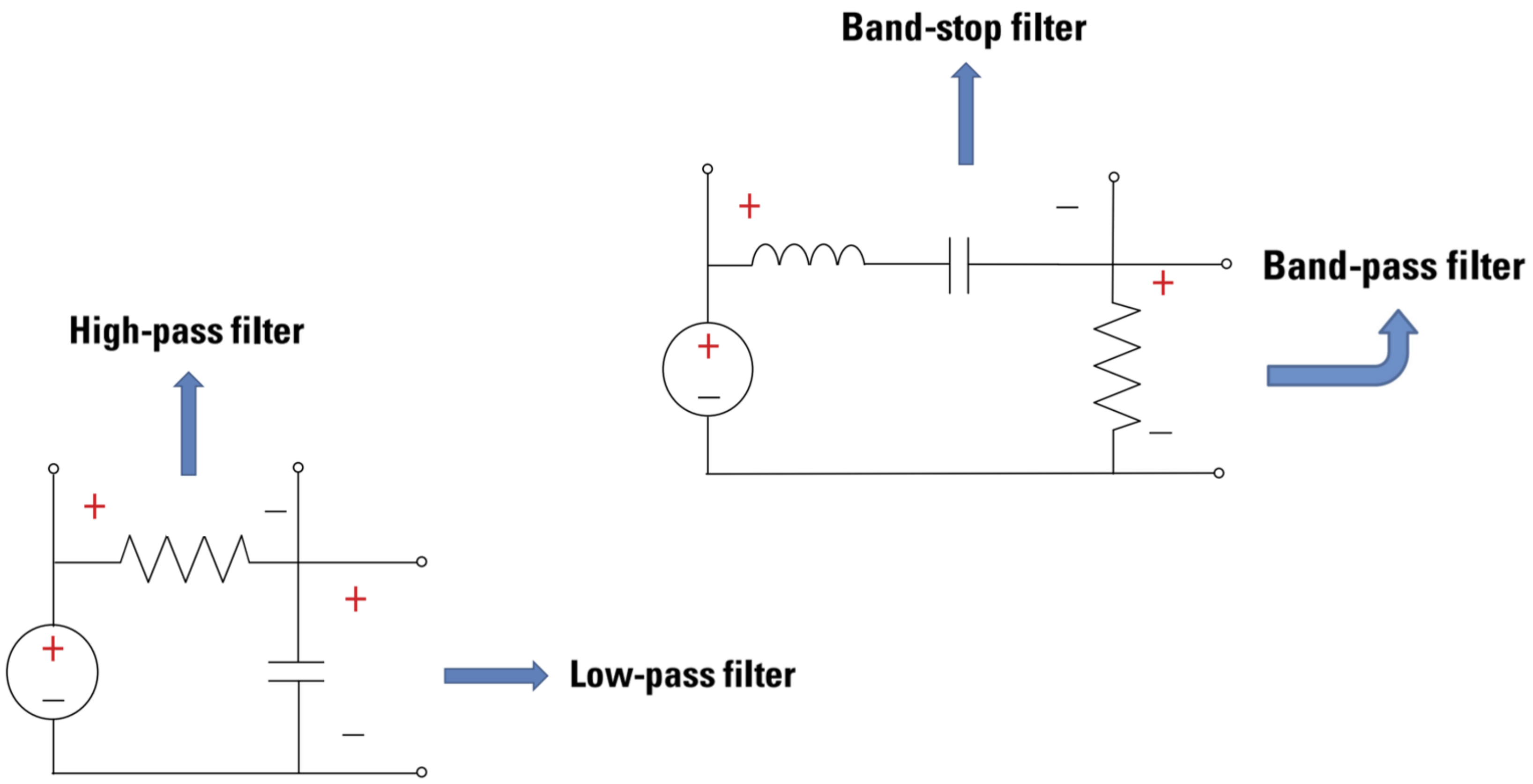
Audio Systems with Multiple Speakers





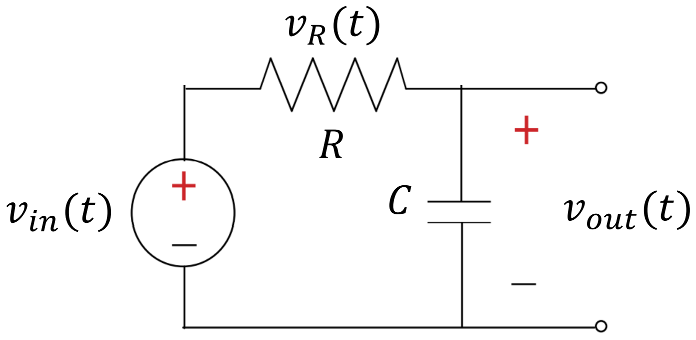
Picture Source: https://www.bhphotovideo.com/explora/amp/audio/tips-and-solutions/what-about-all-those-speaker-specs
Note: This slide has been used in assignment 9 to present applications of Fourier analysis.

Filters Circuits



Picture Source: Santiago, J. (2013). Circuit Analysis for Dummies: John Wiley & Sons. (with modifications)

Low-pass Filter Analysis



All initial conditions are zero.

$$v_{in}(t) = v_R(t) + v_{out}(t)$$

$$v_{in}(t) = Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\, d\tau$$

$$v_{in}(t) = RC\frac{dv_{out}(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} C\frac{dv_{out}(\tau)}{dt}\, d\tau$$

$$v_{in}(t) = RC\frac{dv_{out}(t)}{dt} + v_{out}(t)$$

Picture Source: Santiago, J. (2013). Circuit Analysis for Dummies: John Wiley & Sons. (with modifications)

Low-pass Filter Analysis (cont.)

$$v_{in}(t) = RC\frac{dv_{out}(t)}{dt} + v_{out}(t)$$

Laplace →

$$V_{in}(s) = RCsV_{out}(s) + V_{out}(s)$$

$$V_{in}(s) = V_{out}(s)(RCs + 1)$$

$$V_{out}(s) = \frac{1}{RC}\left(\frac{1}{s + \frac{1}{RC}}\right)V_{in}(s)$$

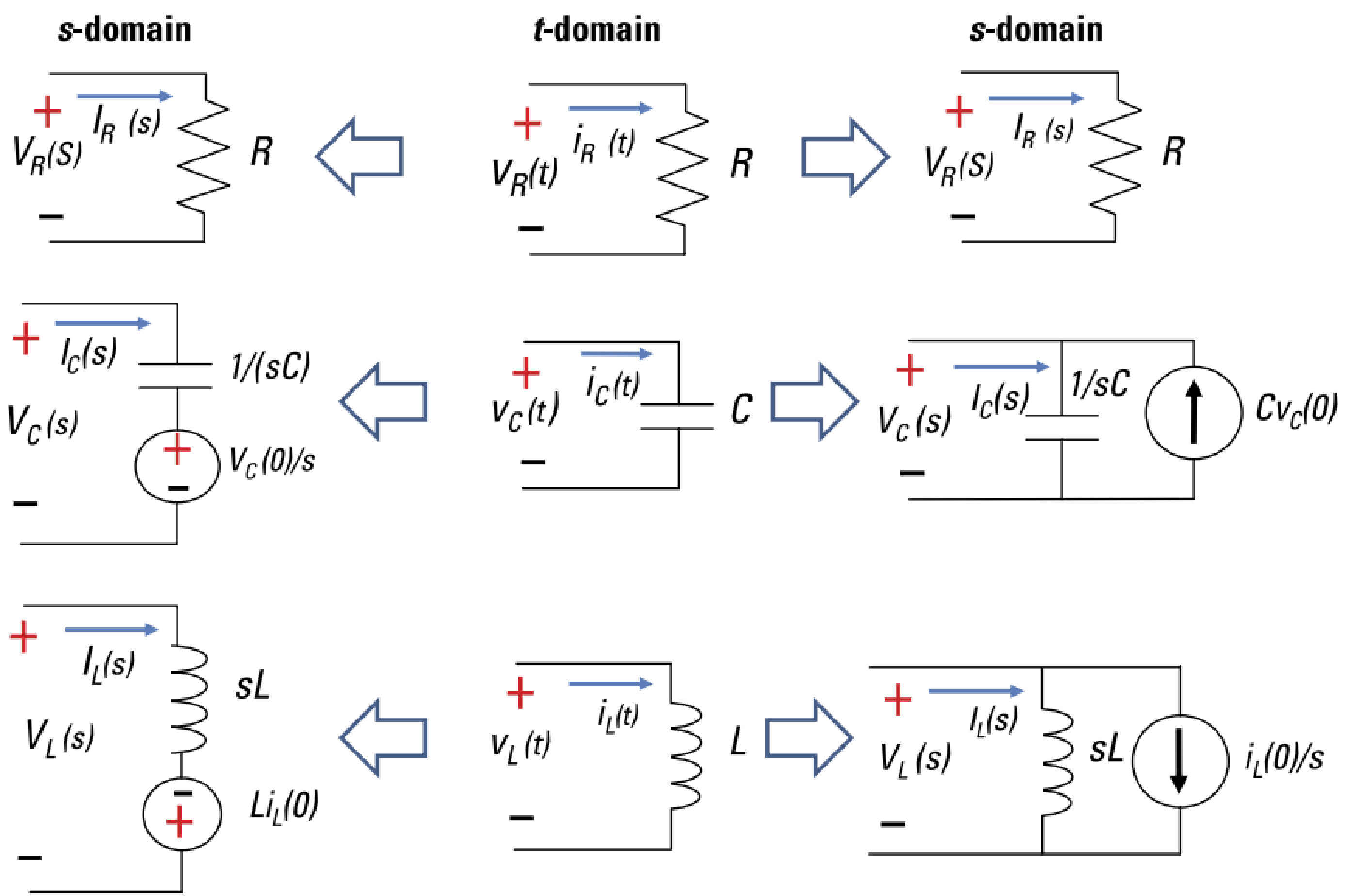
← $Laplace^{-1}$

$$v_{out}(t) = v_{in}(t)(1 - e^{-\frac{1}{RC}t})$$

Devices in S-Domain

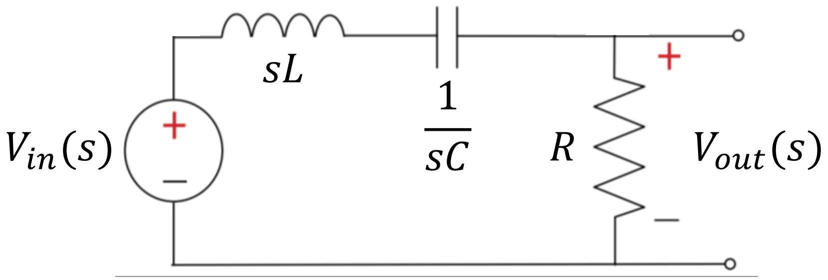
$Device$	$Time-Domain$	$S-Domain$		
		$Voltage$	$Current$	$Impedance$ ($with\ zero\ initial\ condition$)
IVS	$v_S(t)$	$V_S(s)$	$-$	$-$
ICS	$i_S(t)$	$-$	$I_S(s)$	$-$
$VCVS$	$v_2(t)=\mu v_1(t)$	$V_2(s)=\mu V_1(s)$	$-$	$-$
$VCCS$	$i_2(t)=gv_1(t)$	$-$	$I_2(s)=gV_1(s)$	$-$
$CCVS$	$v_2(t)=ri_1(t)$	$V_2(s)=rI_1(s)$	$-$	$-$
$CCCS$	$i_2(t)=\beta i_1(t)$	$-$	$I_2(s)=\beta I_1(s)$	$-$
$Resistor$	$v_R(t)=Ri_R(t)$	$V_R(s)=RI_R(s)$	$I_R(s)=\left(\frac{1}{R}\right)V_R(s)$	$Z_R(s)=R$
$Capacitor$	$v_C(t)=\int_0^t i_C(\tau)\,d\tau$	$V_C(s)=\frac{1}{sC}I_C(s)+\frac{v_{c(0)}}{s}$	$I_C(s)=(sC)V_C(s)-Cv_c(0)$	$Z_C(s)=\frac{1}{sC}$
$Inductor$	$v_L(t)=L\frac{di_L(t)}{dt}$	$V_L(s)=sLI_L(s)-Li_L(0)$	$I_L(s)=\left(\frac{1}{sL}\right)V_L(s)+\frac{i_L(0)}{s}$	$Z_L(s)=sL$

S-Domain Thévenin's and Norton's Equivalent for Passive Elements



Picture Source: Santiago, J. (2013). Circuit Analysis for Dummies: John Wiley & Sons. (with modifications)

Band-pass Filter: S-Domain Analysis



All initial conditions are zero.

$$V_{out}(s) = \left(\frac{R}{sL + \frac{1}{sC} + R}\right) V_{in}(s)$$

$$T(s) = \frac{V_{out}(s)}{V_{in}(s)} = \left(\frac{R}{L}\right) \frac{s}{\left[s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}\right]}$$

Picture Source: Santiago, J. (2013). Circuit Analysis for Dummies: John Wiley & Sons. (with modifications)

Band-pass Filter: S-Domain Analysis (cont.)

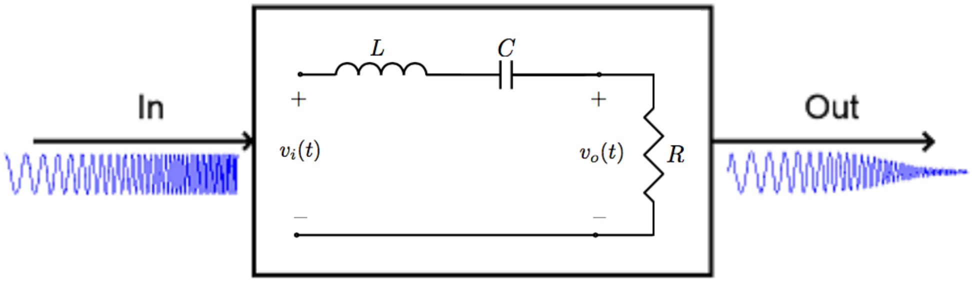
$$s = j\omega \rightarrow T(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \left(\frac{R}{L}\right)\frac{j\omega}{\left[(j\omega)^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}\right]} = \left(\frac{R}{L}\right)\frac{j\omega}{\left[\left(\frac{1}{LC} - \omega^2\right) + \left(\frac{R}{L}\right)j\omega\right]}$$

$$\frac{1}{LC} - \omega^2 = 0 \rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\frac{1}{LC} - \omega^2 = \pm\frac{R}{L}\omega \rightarrow \omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC} = 0 \rightarrow \begin{cases} \omega_{C1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\ \omega_{C2} = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \end{cases}$$

$$Band\_width = \omega_{C2} - \omega_{C1} = \frac{R}{L} \quad and \quad Q\_factor = \frac{\omega_0}{BandWidth} = \frac{1/\sqrt{LC}}{R/L} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Band-pass Filter as a System

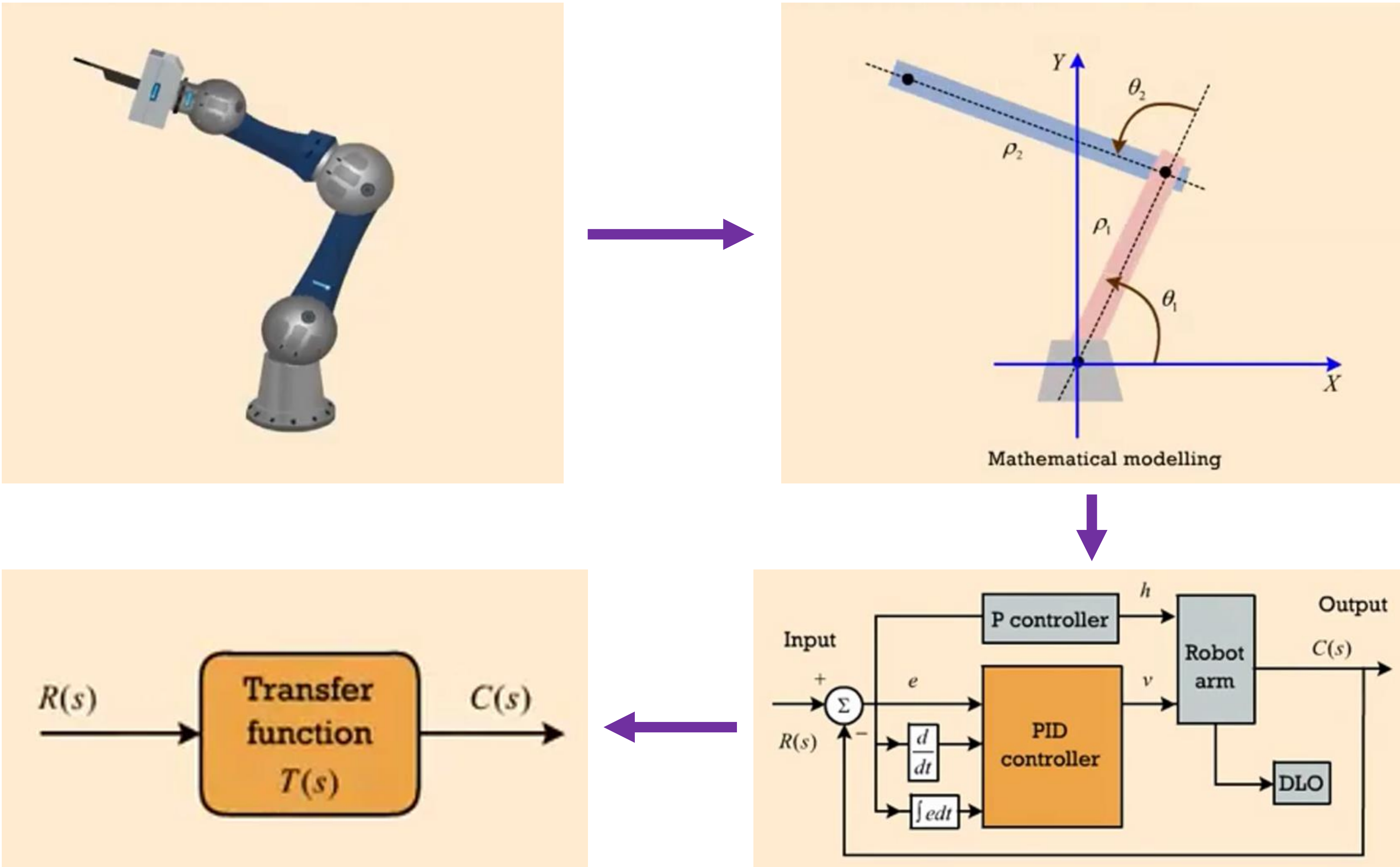


- For a band-pass filter (all initial conditions are zero):

$$T(s) = \frac{A_0 2\zeta\omega_0 s}{s^2 + 2\zeta\omega_0 s + {\omega_0}^2} = \frac{A_0 \frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q} s + {\omega_0}^2} = \frac{A_0 B s}{s^2 + Bs + {\omega_0}^2}$$

A_0: $Midband\ Gain$
ω_0: $Resonant\ Frequency$
ζ: $Damping\ Coefficient$
Q: $Quality\ Factor$

Control Systems and S-Domain



Picture Source: “Applications of Laplace Transform in Control Systems.”, “Mobile Tutor” channel on YouTube.

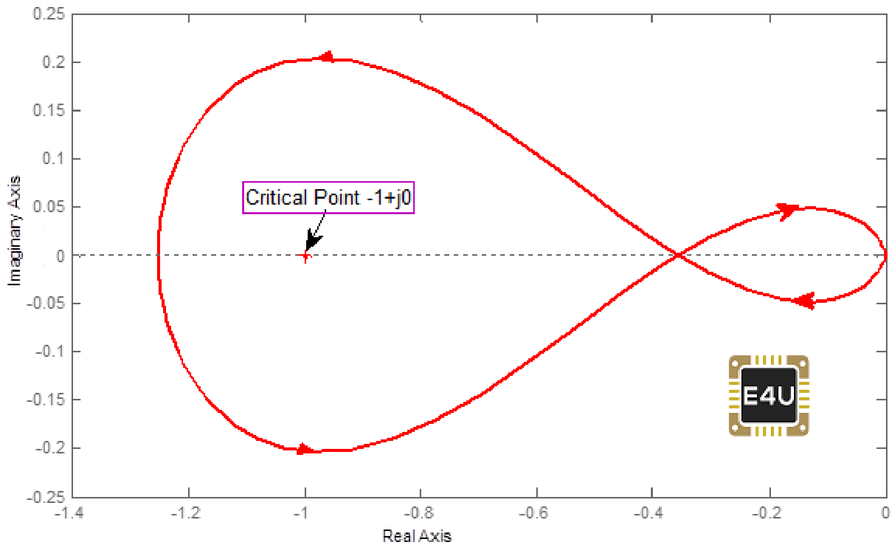
Control Systems and S-Domain (cont.)

- Help in solving differential equations of higher orders and evaluating system output.

- Gain Factor K:

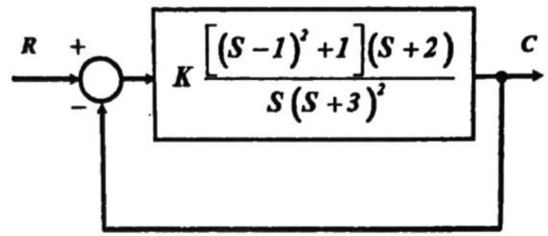
$$T(s) = \frac{1}{As^2 + Bs + C} \xrightarrow{Magnitude\ Amplification} T(s) = K\frac{1}{As^2 + Bs + C}$$

- Nyquist Diagram



Control Systems and S-Domain (cont.)

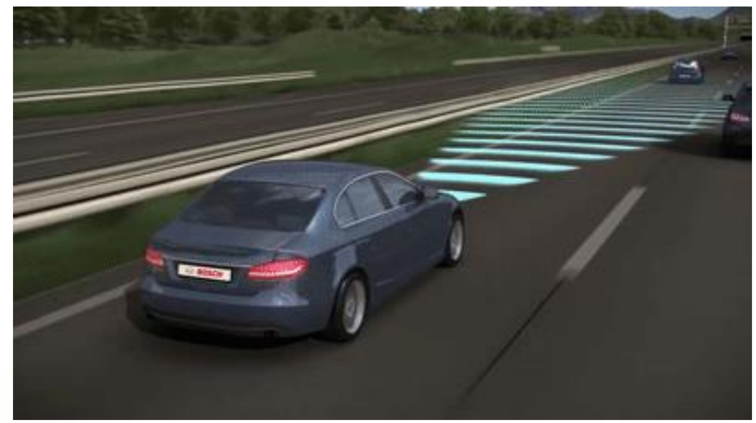
۱- محدوده‌ای برای K طوری بدست آورید که تنها دو قطب حلقه بسته‌ی سیستم زیر بین خطوط $\sigma=0$ و $\sigma=-1$ قرار بگیرند. حداقل مقدار خطای حالت دائم به ورودی شیب واحد و نیز کمترین زمان نشست دو درصد این سیستم در پاسخ به ورودی پله در محدوده‌ی تعیین شده تقریباً چقدر است؟

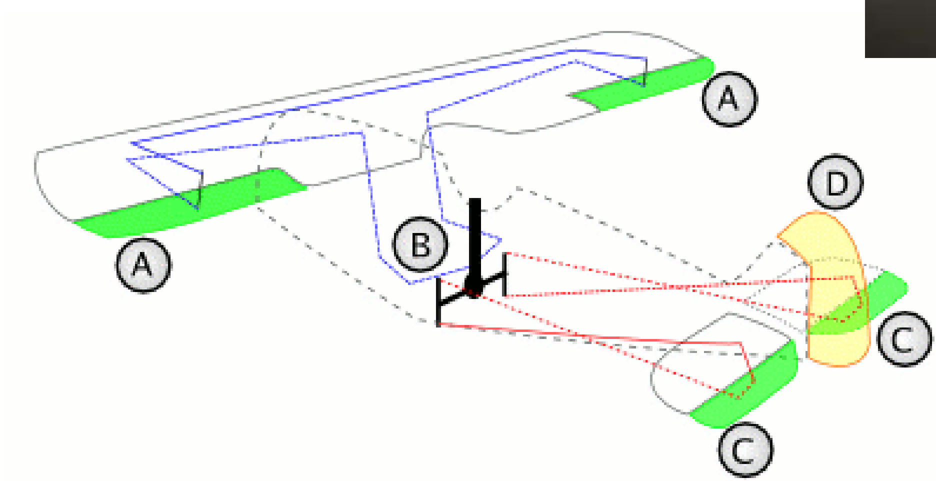


Picture Source: 2010218 (Linear System and Control) at University of Sistan and Baluchestan, Semester 3981, Midterm Exam, by Dr. Saeed Tavakoli Afshari.

Control Systems and S-Domain (cont.)

- Faster forecasting of the outputs of a system.





Picture Source: (1) https://en.m.wikipedia.org/wiki/Aircraft_flight_control_system (2) https://lancmoms.com/adaptive-cruise-control/

The END!